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Discussion of "Objective Priors: An Introduction for Frequentists" by M. Ghosh

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The paper by Ghosh provides a useful introduction to the main ideas underlying objective priors and how these ideas might profitably be used by frequentist statisticians, both at a theoretical and practical level. The aspects likely to be of most interest to this group of statisticians are those concerning probability matching, allowing valid frequentist procedures to be derived via a formal Bayesian analysis. But they should also be interested in priors that arise from decision-theoretic considerations, not least since the consideration of risk criteria, such as mean squared error for estimation or operating characteristic function for testing, is ubiquitous in the frequentist approach. As pointed out by the author, at a theoretical level the shrinkage argument, which I have also used extensively in the past, provides a neat way of deriving frequentist asymptotic results.

My discussion will focus on an examination of the main criteria that have been used to obtain objective priors and, partly related to this, the extent to which the theory and practical application can be extended to more complex scenarios. Before launching into this I would just like to comment on the commonly used term "objective" in the present context. As soon becomes apparent in this field, there is an array of possible criteria available for the development of objective priors, some of which depend on a specific choice of parameterization, and there may be no unique solution even for a given criterion. Thus the choice quickly ceases to be purely objective. My own preference is to use the term "nonsubjective," which indicates that the prior is detached from subjective beliefs about parameters but which does not impart such a strong sense of broad agreement as to what the prior should be in any particular case.

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1. COMPARISON OF CRITERIA

First, a general point about alternative criteria for the development of objective priors. I have a strong preference for criteria that would lead to the use of properly calibrated subjective priors whenever they are available, so that the consideration of objective priors in some sense generalizes a property of a fully subjective Bayesian approach. In a sense this is true of probability matching since this leads to (approximately) correct coverage of posterior regions in hypothetical repeated sampling. This in turn implies that these regions will also be calibrated over repeated *use*, as would automatically be the case if a properly elicited subjective prior were to be used. The same cannot be said for moment matching in the sense described in Section 5.2; there seems nothing in this criterion that would lead one to use a subjective prior when available.

Similarly, consideration of a proper scoring rule in a decision-theoretic approach would indicate the use of an elicited subjective prior whenever one is available. As a consequence, I would be uneasy using a decision-theoretic criterion that was not based on a proper scoring rule. For example, it does seem surprising that, even in the scalar parameter case, Jeffreys' prior turns out not to be optimal under the distance measure (3.13) with $\beta = -1$. The problem is that, unlike the Bernardo criterion that arises when $\beta = 0$ (see later), none of these distance measures corresponds to an average regret based on some primitive loss function that produces a (negative) score when data x are observed and a prior predictive distribution $\pi(x)$ is adopted. So there seems to be no obvious sense in which we would recover a subjective prior distribution whenever one is available.

Although there is some reference to predictive probability matching in Sections 5 and 6, the paper is largely a review of objective priors obtained via parametric criteria, which usually require a focus on one or more specified parameters of interest. This has certainly been the most popular area of study and, as