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Rejoinder

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Abstract. I thank the editor for the opportunity to expand upon the paper, and I thank the discussants for their insightful comments. In this rejoinder I elaborate on some of the topics from the discussion: the appropriateness of separable covariance models for array-valued data, the role of priors and penalties on estimation and the limiting nature of array valued data.

The article and discussion provide several examples of array-valued and matrix-valued data, including relational (network) data, space-time and imaging data. Additionally, in many statistical models the parameters themselves are arrays, even though the data are not. For example, consider a three-factor experiment or study in which the levels of the factors are indexed by the sets $\mathcal{I}, \mathcal{J}, \mathcal{K}$. Letting $y_{i,j,k,l}$ be the measurement on the lth subject with levels i, j and k of the three factors, the data itself may not be an array as the number of subjects per factor combination may vary, but the unknown cell means $\{\mu_{i,j,k} : i \in \mathcal{I}, j \in \mathcal{J}, k \in \mathcal{K}\}$ constitute an array of dimension $|\mathcal{I}| \times |\mathcal{J}| \times |\mathcal{K}|$.

It is often desirable to estimate or account for patterns of dependence or similarity among objects in the index sets of such arrays. The article provides some computational tools for doing so, by relating the multilinear Tucker product to a class of multivariate normal distributions with separable covariance structure. A separable covariance structure is a "reduced model", in the sense that not all covariance matrices are separable. In what situations is such a model restriction justifiable? What are the alternatives?

Lopes expresses some concern that separability might not be an appropriate assumption for space-time data. Indeed, Stein (2005) makes a convincing argument against using separable covariance matrices for such applications. For space-time data, however, we can often rely on some degree of smoothness or continuity. Smoothness in time and space allows us to build rich but relatively parsimonious dependence models based on a small number of parameters that describe spatial or temporal correlation functions. Thus in the space-time domain, there are a large number of non-separable alternatives to modeling dependence patterns. Even so, in some situations separability may still be useful: Genton (2007) argues that separable approximations to non-separable covariance matrices can be useful for some inferential tasks. Additionally, judicious combinations of separable and non-separable structure lead to flexible models as in Lopes et al. (2008). Another way to use separable covariance structure for non-separable covariance estimation is in prior specification: Consider a non-separable covariance matrix $\Sigma = \text{Cov}[\text{vec}(\mathbf{Y})]$, where $\mathbf{Y}$ is a multiway array. A hierarchical prior for Σ could be of the form $\Sigma^{-1} \sim \text{Wishart}(\nu_0, c \times \Sigma_K \otimes \cdots \otimes \Sigma_1)$, with the Σ_k's also having inverse-Wishart priors. This centers the prior for Σ around a separable value, but Σ is non-separable with probability one.

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