

CORRECTION
ERROR ESTIMATES FOR BINOMIAL APPROXIMATIONS
OF GAME OPTIONS

BY YURI KIFER

The Annals of Applied Probability **16** (2006) 984–1033

My student Y. Dolinsky noticed that the inequalities (5.33) needed to obtain the estimate (5.34) in the proof of Theorem 2.3 hold true only for hedging strategies without short selling of bonds and stocks, that is, when the amounts of bonds and stocks in the portfolio are always nonnegative. Since the existence of such hedging strategies cannot be guaranteed, in general, if their initial capital equals the option price, the proof should be corrected and we start with an argument due to Dolinsky which serves this purpose. In the notation of [1] set

$$(1) \quad \Psi = \sup_{0 \leq t \leq T} \left(Q_z^B(\theta_\varphi^{(n)}, t) - Q_z^{B,n} \left(\frac{\varphi T}{n}, \frac{\nu_t T}{n} \right) \right)^+.$$

From (5.29)–(5.32) of [1], we obtain that there exists a constant $C > 0$ such that

$$(2) \quad E^B \Psi \leq C(F_0(z) + \Delta_0(z) + z + 1)n^{-1/4}.$$

Let $\tau \in \mathcal{T}_{0T}^B$ be a stopping time. Then $\nu_\tau = \min\{k \in \mathbb{N} : \theta_k^{(n)} \geq \tau\} \in \mathcal{T}^{B,n}$, and so $\theta_{\nu_\tau}^{(n)} \in \mathcal{T}^B$ is a stopping time (see beginning of proofs of Lemmas 3.1 and 3.6 in [1]). As any self-financing discounted portfolio \check{Z}^B (see (5.21) in [1]) is a martingale, and so taking into account (5.24) in [1] and the optional sampling theorem, we derive that

$$(3) \quad \begin{aligned} \check{Z}_{\theta_\varphi^{(n)} \wedge \tau}^B &= E^B(\check{Z}_{\theta_\varphi^{(n)} \wedge \theta_{\nu_\tau}^{(n)}}^B | \mathcal{F}_{\theta_\varphi^{(n)} \wedge \tau}) \\ &\geq E^B \left(Q_z^{B,n} \left(\frac{\varphi T}{n}, \frac{\nu_\tau T}{n} \right) \middle| \mathcal{F}_{\theta_\varphi^{(n)} \wedge \tau} \right) \\ &\geq E^B(Q_z^B(\theta_\varphi^{(n)}, \tau) - \Psi | \mathcal{F}_{\theta_\varphi^{(n)} \wedge \tau}) \\ &= Q_z^B(\theta_\varphi^{(n)}, \tau) - E^B(\Psi | \mathcal{F}_{\theta_\varphi^{(n)} \wedge \tau}). \end{aligned}$$

Finally, (3) yields that

$$\sup_{\tau \in \mathcal{T}_{0T}^B} E^B(Q_z^B(\theta_\varphi^{(n)}, \tau) - \check{Z}_{\theta_\varphi^{(n)} \wedge \tau}^B)^+ \leq E^B \Psi,$$

Received October 2006.