

Isaac Newton as a Probabilist

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Abstract. In 1693, Isaac Newton answered a query from Samuel Pepys about a problem involving dice. Newton's analysis is discussed and attention is drawn to an error he made.

On November 22, 1693, Samuel Pepys wrote a letter to Isaac Newton posing a problem in probability. Newton responded with three letters, first answering the question briefly, and then offering more information as Pepys pressed for clarification. Pepys (1633–1703) is best known today for his posthumously published diary covering the intimate details of his life over the years 1660–1669, but Newton would not have been aware of that diary. He would instead have known of Pepys as a former Secretary of Admiralty Affairs who had served as President of the Royal Society of London from 1684 through November 30, 1686, the same period when Newton's great *Principia* was presented to the Royal Society and its preparation for the press begun. But Pepys' letter did not concern scientific matters. He sought advice on the wisdom of a gamble.

1. PEPYS' PROBLEM

The three letters Newton wrote to Pepys on this problem, on November 26 and December 16 and 23, 1693, are almost all we have bearing on Newton and probability. Some of the letters were published with other private correspondence in Pepys (1825, Vol. 2, pages 129–135; 1876–1879, Vol. 6, pages 177–181) and more completely in Pepys (1926, Vol. 1, pages 72–94). The letters were cited in a textbook by Chrystal (1889, page 563), where he gave Pepys' problem as an exercise, but they were little known until they were brought to a wide public attention when selections were reprinted with commentary independently by Dan Pedoe (1958, pages 43–48), Florence David (1959; 1962, pages 125–129) and Emil D. Schell (1960). These authors and several others, notably Chaundy and Bullard (1960), Mosteller (1965, pages 6, 33–35) and

Gani (1982) have discussed the problem Pepys posed and Newton's solution. Others accorded it briefer notice, including Sheynin (1971), who dismissively relegated it to a footnote; Westfall (1980, pages 498–499), who gave unwarranted credence to the excuse Pepys opened his first letter with, that the problem had some connection to a state lottery; and Gjertsen (1986, pages 427–428). But none of these or any other writer seems to have noted that a major portion of Newton's solution is wrong. The error casts an interesting light on how Newton thought about the matter, and it seems useful to revisit the question.

Since Pepys' original statement was, as Newton noticed, somewhat ambiguous, I will state the problem in paraphrase as it emerged in the correspondence:

Which of the following three propositions has the greatest chance of success?

- A. Six fair dice are tossed independently and at least one "6" appears.
- B. Twelve fair dice are tossed independently and at least two "6"s appear.
- C. Eighteen fair dice are tossed independently and at least three "6"s appear.

As it emerged in the correspondence, Pepys initially thought that the third of these (C) was the most probable, but when Newton convinced him after repeated questioning by Pepys that in fact A was the most probable, Pepys ended the correspondence and announced he would, using Mosteller's (1965, page 35) colorful later term, *welsh on a bet* he had made.

2. NEWTON'S SOLUTION

Newton stated the solution three times during the correspondence: first he gave a simple logical reason for concluding that A is the most probable, then he reported a detailed exact enumeration of the chances in each of the three cases, and finally he returned to the logical argument and gave it in more detail.

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