

CORRECTION

SDEs WITH OBLIQUE REFLECTIONS ON NONSMOOTH DOMAINS

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It has been pointed out by Weining Kang and Ruth Williams that there is an error in an argument in [1]. The purpose of this note is to correct the argument.

The error affects only Case 2 of the paper, and occurs in the first display at the top of page 580, at the end of the proof of Theorem 5.1. This display claims that a certain bound follows from (3.28) of the paper, and implicitly assumes that if

$$(p, q) \doteq Df_\varepsilon^\beta(Y(s), Y'(s))$$

and if $Y(s) \in \partial G$ and $Y'(s) \in \overline{G}$, then $Y(s) + \beta p \in \partial G$ and $Y'(s) + \beta q \in \overline{G}$, which need not be true. The statement of the theorem is still correct, and the reason is that underlying assumptions are in some sense robust with respect to small perturbations of the boundary.

Before presenting the correction we review the assumption made in Case 2. There is an open set W containing \overline{G} and a $C^{2,+}$ function g on $W \times \mathbb{R}^N$ which for each fixed $x \in W$ is C^1 as a mapping $r \mapsto g(x, r)$. Furthermore there are constants $C > 0$ and $\theta > 0$ such that

$$(3.21) \quad g(x, 0) = 0,$$

$$(3.22) \quad g(x, r) \geq |r|^2,$$

$$(3.23) \quad \langle D_r g(x, r), \gamma_i(x) \rangle \geq 0 \quad \text{if } \langle r, n_i(x) \rangle \geq -\theta|r|,$$

$$(3.24) \quad |p| \leq C|r|^2, \quad |q| \leq C|r| \quad \text{if } (p, q) \in D^+g(x, r),$$

and for any $x \in \overline{G}$, $r \in \mathbb{R}^N$, there is $(p, q) \in D^+g(x, r)$ such that

$$(3.25) \quad \left((p, q), C \begin{pmatrix} |r|^2 I & 0 \\ 0 & I \end{pmatrix} \right) \in D^{2,+}g(x, r).$$

A more careful statement of condition (3.23) is

$$\langle D_r g(x, r), \gamma_i(x) \rangle \geq 0 \quad \text{if } x \in \partial G, i \in I(x) \text{ and } \langle r, n_i(x) \rangle \geq -\theta|r|.$$

The next lemma shows that (3.23) is in some sense robust.

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