The Annals of Probability
2006, Vol. 34, No. 1, 423–426
DOI: 10.1214/009117905000000503

CORRECTION

IMPROPER REGULAR CONDITIONAL DISTRIBUTIONS

BY TEDDY SEIDENFELD, MARK J. SCHERVISH AND JOSEPH B. KADANE

Carnegie Mellon University

A strict inequality appears in Definition 6 where a weak inequality is needed. We reproduce Definition 6 here.

DEFINITION 6. Fix ω and consider those A such that $\omega \in A \in \mathcal{A}$. If for some $\omega \in A \in \mathcal{A}$, $P(A|\mathcal{A})(\omega) = 0$, say that $P(\cdot|\mathcal{A})$ is *maximally improper at* ω. Otherwise, if for each $\omega \in A \in \mathcal{A}$, $1 \geq P(A|\mathcal{A})(\omega) > 0$, say that the rcd is *modestly proper at* ω.

At the bottom of page 1614, we are not precise in the definition of a Borel space. The condition should have read that there is a one-to-one measurable function with measurable inverse between $(\Omega, \mathcal{B})$ and $(E, \mathcal{E})$, where E is a Borel subset of the reals and $\mathcal{E}$ is the Borel σ-field of subsets of E. After the remaining corrections below, our use of the term “Borel space” conforms with this definition.

Some conditions were left out of Theorem 4 and Lemma 3. The proof of Lemma 3 also had some errors that made it almost impossible to follow. Finally, the proof of Theorem 4 was said to be straightforward from Theorem 3. We include here the restatements of both results with the missing conditions, the revised proof of Lemma 3, and a proof of Lemma 4. The only application of Lemma 4 given in the original paper is to the proof of Corollary 2. The additional conditions given here are satisfied in that case.

THEOREM 4. *Assume that $\mathcal{A}$ is an atomic sub-σ-field of $\mathcal{B}$. Let $(\Theta, \mathcal{D})$ be a Borel space, with a probability measure μ. For each $\theta \in \Theta$, let P_θ be a probability on $\mathcal{B}$ such that for every $B \in \mathcal{B}$, $P_\theta(B)$ is a $\mathcal{D}$-measurable function of θ. Let $P(\cdot)$ be defined on $\mathcal{B}$ by $P(\cdot) = \int_\Theta P_\theta(\cdot)\,d\mu(\theta)$. Assume that, for μ-almost all θ, $P_\theta(\cdot|\mathcal{A})$ is a maximally improper rcd for P_θ and that it is $\mathcal{A} \otimes \mathcal{D}$-measurable as a function of (ω, θ). Also, assume that the set*

$$B^* = \{(\omega, \theta) : P_\theta(\cdot|\mathcal{A}) \text{ is maximally improper at } \omega\},$$

is in $\mathcal{A} \otimes \mathcal{D}$. Then there is a maximally improper version of $P(\cdot|\mathcal{A})$.

Received October 2004; revised April 2005.