ON MODULI OF k-CONVEXITY

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Received 11 October 1999

We establish the continuity of some moduli of k-convexity. Let X be a Banach space. We denote by X^* the dual space of X and by B_X the unit ball of X. Several moduli of convexity for the norm of X have been defined; the last two definitions in the following are valid for spaces having dimension $\geq k$:

$$\delta_X(\epsilon) = \inf\left\{1 - \frac{\|x+y\|}{2} : x, y \in B_X, \|x-y\| \geq \epsilon\right\} \quad \text{(see [2])},$$
$$\delta_X^{(k)}(\epsilon) = \inf\left\{1 - \frac{\|x_1+\cdots+x_{k+1}\|}{k+1} : x_1, \ldots, x_{k+1} \in B_X, A\big(x_1, \ldots, x_{k+1}\big) \geq \epsilon\right\} \quad \text{(see [10])},$$
$$\Delta_X^{(k)}(\epsilon) = \inf_{\|x\|=1} \inf_{\substack{Y \subset X \\ \dim(Y)=k}} \sup_{\substack{\|y\|=1 \\ y \in Y}} \big\{\|x+\epsilon y\| - 1\big\} \quad \text{(see [9])}, \tag{1}$$

where

$$A\big(x_1, \ldots, x_{k+1}\big) = \frac{1}{k!} \sup\left\{\begin{vmatrix} 1 & \cdots & 1 \\ f_1\big(x_1\big) & \cdots & f_1\big(x_{k+1}\big) \\ \vdots & \cdots & \vdots \\ f_k\big(x_1\big) & \cdots & f_k\big(x_{k+1}\big) \end{vmatrix} : f_1, \ldots, f_k \in B_{X^*}\right\}. \tag{2}$$

Evidently, by subtracting the first column from the other columns, the determinant can be replaced by

$$\begin{vmatrix} f_1\big(x_2 - x_1\big) & \cdots & f_1\big(x_{k+1} - x_1\big) \\ \vdots & \cdots & \vdots \\ f_k\big(x_2 - x_1\big) & \cdots & f_k\big(x_{k+1} - x_1\big) \end{vmatrix}. \tag{3}$$

Also $A(x_1, \ldots, x_{k+1})$ can be thought of as the "volume" of the convex hull of $x_1, \ldots,$ x_{k+1} since that is the case in Euclidean spaces.

Abstract and Applied Analysis 4:4 (1999) 243–247
1991 Mathematics Subject Classification: 46B20, 47H10
URL: http://aaa.hindawi.com/volume-4/S1085337599000202.html