EXISTENCE AND REGULARITY OF WEAK SOLUTIONS TO THE PRESCRIBED MEAN CURVATURE EQUATION FOR A NONPARAMETRIC SURFACE

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Received 15 August 1998

1. Introduction

The prescribed mean curvature equation with Dirichlet condition for a nonparametric surface $X : \Omega \to \mathbb{R}^3$, $X(u,v) = (u,v,f(u,v))$ is the quasilinear partial differential equation

$$\begin{gathered}\left(1+f_v^2\right)f_{uu}+\left(1+f_u^2\right)f_{vv}-2f_uf_vf_{uv}=2h(u,v,f)\left(1+|\nabla f|^2\right)^{3/2} \quad \text{in } \Omega,\\ f=g \quad \text{in } \partial\Omega,\end{gathered} \tag{1.1}$$

where Ω is a bounded domain in $\mathbb{R}^2$, $h : \overline{\Omega}\times\mathbb{R}\to\mathbb{R}$ is continuous and $g \in H^1(\Omega)$.

We call $f \in H^1(\Omega)$ a weak solution of (1.1) if $f \in g + H_0^1(\Omega)$ and for every $\varphi \in C_0^1(\Omega)$

$$\int_\Omega \left(\left(1+|\nabla f|^2\right)^{-1/2}\nabla f\nabla\varphi + 2h(u,v,f)\varphi\right)du\,dv = 0. \tag{1.2}$$

It is known that for the parametric Plateau's problem, weak solutions can be obtained as critical points of a functional (see [2, 6, 7, 8, 10, 11]).

The nonparametric case has been studied for $H = H(x,y)$ (and generally $H = H(x_1,\dots,x_n)$ for hypersurfaces in $\mathbb{R}^{n+1}$) by Gilbarg, Trudinger, Simon, and Serrin, among other authors. It has been proved [5] that there exists a solution for any smooth boundary data if the mean curvature H' of $\partial\Omega$ satisfies

$$H'\left(x_1,\dots,x_n\right) \ge \frac{n}{n-1}\left|H\left(x_1,\dots,x_n\right)\right| \tag{1.3}$$

for any $(x_1,\dots,x_n) \in \partial\Omega$, and $H \in C^1(\overline{\Omega},\mathbb{R})$ satisfying the inequality

$$\left|\int_\Omega H\varphi\right| \le \frac{1-\epsilon}{n}\int_\Omega |D\varphi| \tag{1.4}$$

Abstract and Applied Analysis 4:1 (1999) 61–69
1991 Mathematics Subject Classification: 35J20, 35J65
URL: http://aaa.hindawi.com/volume-4/S1085337599000019.html