THE EFFECT OF THE GRAPH TOPOLOGY ON THE EXISTENCE OF MULTIPEAK SOLUTIONS FOR NONLINEAR SCHRÖDINGER EQUATIONS

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1. Introduction

Consider

$$\begin{cases} -\varepsilon^2\Delta u + V(y)u = u^{p-1}, & y \in R^N, \\ u > 0, & y \in R^N, \\ u \to 0, & \text{as } |y| \to +\infty, \end{cases} \tag{1.1}$$

where $V(y)$ is a smooth bounded function with positive lower bound, $\varepsilon > 0$ is a small number, $2 < p < \frac{2N}{N-2}$ if $N > 2$ and $2 < p < +\infty$ if $N = 2$.

Many works have been done on problem (1.1) recently. See for example [6, 7, 8, 16, 21, 22, 23]. One of the results in the papers just mentioned is that if $x_1, x_2, \ldots, x_k$ are k different strictly local minimum points of $V(y)$, then (1.1) has a k-peak solution u_ε, that is, solution with exactly k local maximum points, such that u_ε has exactly one local maximum point in a neighbourhood of x_j, $j = 1, \ldots, k$. The same conclusion is also true if $x_1, x_2, \ldots, x_k$ are k different strictly local maximum points of $V(y)$. Actually, it is proved in [23] that (1.1) has a multipeak solution with all its peaks near an isolated maximum point of $V(y)$. Thus a natural question is what will happen if $V(y)$ attains its local minimum or local maximum on a connected set. Especially, if $V(y)$ attains its local minimum on a connected set which contains infinitely many points, it is interesting to study whether (1.1) has a multipeak solution concentrating on this set. Generally, this is not true as shown by example (1.6).

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