EXISTENCE OF POSITIVE ENTIRE SOLUTIONS OF SEMILINEAR ELLIPTIC EQUATIONS ON $\mathbf{R^N}$

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1. Introduction

In the present paper we are concerned with positive solutions of the following problem:

$$(P) \qquad \begin{cases} -\Delta u + u = g(x,u), \quad x \in R^N, \\ u \in H^1(R^N), \quad N \geq 3, \end{cases}$$

where $g : R^N \times R \to R$ is a continuous mapping. Recently, the existence of positive solutions of the semilinear elliptic problem

$$(P_Q) \qquad \begin{cases} -\Delta u + u = Q(x) \mid u \mid^{p-1} u, \quad x \in R^N, \\ u \in H^1(R^N), \quad N \geq 2, \end{cases}$$

has been studied by several authors, where $1 < p$ for $N = 2$, $1 < p < (N+2)/(N-2)$ for $N \geq 3$ and $Q(x)$ is a positive bounded continuous function. If $Q(x)$ is a radial function, we can find infinity many solutions of problem (P_Q) by restricting our attention to the radial functions (cf. [2, 5]). If $Q(x)$ is nonradial, we encounter a difficulty caused by the lack of a compact embedding of Sobolev type. To overcome this kind of difficulty, P. L. Lions developed the concentrate compactness method [8, 9], and established the following result: Assume that $lim_{|x|\to\infty} Q(x) = \overline{Q}(> 0)$ and $Q(x) \geq \overline{Q}$ on R^N. Then the problem (P_Q) has a positive solution. This result is based on the observation that the ground state level c_Q of the functional

$$I_Q(u) = \frac{1}{2}\int_{R^N} (\mid \nabla u \mid^2 + \mid u \mid^2) dx - \frac{1}{p+1}\int_{R^N} Q(x) \mid u \mid^{p+1} dx$$

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