SINGULAR ESTIMATES AND UNIFORM STABILITY OF COUPLED SYSTEMS OF HYPERBOLIC/PARABOLIC PDES

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1. Statement of the problem. Main results

1.1. Introduction; singular estimates for coupled PDE systems. The present paper sets itself along that line of research established by [1, 3]—and later streamlined and simplified in the exposition of [16]—that is focused on mathematical properties of an accepted acoustic chamber model [8, 31], subject to unbounded control action on its flexible wall.

We consider a generalization of an established structural acoustic model. This consists of a coupled system of two partial differential equations of different types—a hyperbolic PDE acting within the acoustic chamber and a parabolic-like PDE acting on the elastic wall of the chamber—which are strongly coupled at their common interface. Unlike prior literature, we allow the parameter α—which measures the strength of damping in the plate-like component—to run over the entire range $1/2 \le \alpha \le 1$ of analyticity (parabolicity) of its free dynamics. Prior literature considered only the limit case $\alpha = 1$ ("Kelvin Voight damping"). However, the limit case $\alpha = 1/2$ ("structural, square root damping") is at least equally important; indeed, even more so in applications. Entirely new phenomena arise over the case $\alpha = 1$ as α decreases from 1 to 1/2, which we describe below. Indeed, as α decreases from 1 to 1/2, we seek to achieve two critical control-theoretic properties of the overall coupled system: (i) a quantitatively precise singular estimate for the operator $e^{At}B$ as $t \searrow 0$ and, simultaneously, (ii) a uniform (exponential) stability result of the free dynamics operator e^{At} (which is *not* analytic, of course). Here, B is the unbounded control operator: it typically contains derivatives of Dirac measures supported on points or curves of the elastic wall of the chamber. It turns out that goals (i) and (ii) are in conflict with one another, as α decreases from 1 to 1/2. Indeed, as the damping strength α of the plate (parabolic) component *decreases* from 1 to 1/2, a progressively *stronger*

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