

# DEVELOPMENT OF LOCAL, GLOBAL, AND TRACE ESTIMATES FOR THE SOLUTIONS OF ELLIPTIC EQUATIONS

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## 1. Introduction

In this paper, we improve and extend the local, global, and trace estimates for the solutions of elliptic equations we developed in [6]. These estimates are developed for the solutions of elliptic second-order equations, with source terms, with Dirichlet-Neumann boundary conditions and Dirichlet boundary conditions. They are obtained under quasi-optimal regularity conditions on the source terms. The previous trace estimates we developed in [6] are based on our improved version of the estimates of Aleksandrov, Bakelman, and Pucci estimates [1, 2]. They are improvement in the sense that they are independent of the low, “relaxation,” term of the equations. These later estimates are important in many applications, in particular, for the treatment of quasilinear and fully nonlinear equations. The trace estimates we developed are themselves of great importance in elliptic theory. They are also important in many other applications, in particular, for the study of fixed point methods, the treatment of contraction properties for some operators, and also for the analysis of algorithms in numerical analysis. We have already given some applications of the trace estimates of [6] in [4].

In Section 2, we state a local estimate for elliptic equations. In Section 3, we introduce the first and second basic problems corresponding, respectively, to elliptic equations with Dirichlet-Neumann and Dirichlet-Dirichlet boundary conditions. In Section 4, we state our trace estimates for the first and second basic problems. In Section 5, we state and prove various local and global estimates for the first basic problem. In Section 6, we prove the trace estimate for the first basic problem. In Section 7, we state and prove various local and global estimates for the second basic problem. In Section 8, we prove the trace estimate for the second basic problem. In Section 9, we introduce the third basic problem corresponding to elliptic equations with Dirichlet-Neumann boundary