THE EFFECT OF THE GRAPH TOPOLOGY ON THE EXISTENCE OF MULTIPEAK SOLUTIONS FOR NONLINEAR SCHRÖDINGER EQUATIONS

E. N. DANCER, KEE Y. LAM, AND SHUSEN YAN

Received 16 September 1998

1. Introduction

Consider the problem

$$\begin{gathered} -\varepsilon^2 \Delta u + V(y)u = u^{p-1}, \quad y \in \mathbb{R}^N, \\ u > 0, \quad y \in \mathbb{R}^N, \\ u \longrightarrow 0, \quad \text{as } |y| \longrightarrow +\infty, \end{gathered} \tag{1.1}$$

where $V(y)$ is a smooth bounded function with positive lower bound, $\varepsilon > 0$ is a small number, $2 < p < 2N/(N-2)$ if $N > 2$ and $2 < p < +\infty$ if $N = 2$.

Many works have been done on problem (1.1) recently (cf. [6, 7, 8, 16, 21, 22, 23]). One of the results in the papers just mentioned is that if $x_1, x_2, \dots, x_k$ are k different strictly local minimum points of $V(y)$, then (1.1) has a k-peak solution u_ε, that is, solution with exactly k local maximum points, such that u_ε has exactly one local maximum point in a neighborhood of x_j, $j = 1, \dots, k$. The same conclusion is also true if $x_1, x_2, \dots, x_k$ are k different strictly local maximum points of $V(y)$. Actually, it is proved in [23] that (1.1) has a multipeak solution with all its peaks near an isolated maximum point of $V(y)$. Thus a natural question is what will happen if $V(y)$ attains its local minimum or local maximum on a connected set. Especially, if $V(y)$ attains its local minimum on a connected set which contains infinitely many points, it is interesting to study whether (1.1) has multipeak solution concentrating on this set. Generally, this is not true as shown in Example 1.6.

The main results of this paper consist of three parts. First, we study how the topological structure of the local minimum set of the potential $V(y)$ affects the existence of multipeak solutions for (1.1). We show that if the minimum set of $V(y)$ has nontrivial reduced homology, then for each $k \geq 1$, (1.1) has at least

Abstract and Applied Analysis 6:2 (2001) 71–99
2000 Mathematics Subject Classification: 35J65, 35B38, 58E05
URL: http://aaa.hindawi.com/volume-6/S1085337501000276.html