EXISTENCE AND NONEXISTENCE OF GLOBAL SOLUTIONS OF DEGENERATE AND SINGULAR PARABOLIC SYSTEMS

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1. Introduction

We study the problem of the existence and nonexistence of global weak solutions of the initial value problem for systems of parabolic inequalities of the following two types:

$$\begin{aligned} u_t - |x|^{\tau_1}\Delta u &\geq |v|^q, \quad (x,t) \in \mathbb{R}^N \times (0,\infty), \\ v_t - |x|^{\tau_2}\Delta v &\geq |u|^p, \quad (x,t) \in \mathbb{R}^N \times (0,\infty), \end{aligned} \tag{1.1}$$

$$\begin{aligned} u_t - \Delta u &\geq t^{k_1}|x|^{-\sigma_1}|v|^q, \quad (x,t) \in \mathbb{R}^N \times (0,\infty), \\ v_t - \Delta v &\geq t^{k_2}|x|^{-\sigma_2}|u|^p, \quad (x,t) \in \mathbb{R}^N \times (0,\infty), \end{aligned} \tag{1.2}$$

where $p,q > 1$ and $u(x,0) = u_0(x)$, $v(x,0) = v_0(x)$, $x \in \mathbb{R}^N$. Systems like (1.1) and (1.2) will be called *degenerate* and *singular*, respectively. Several authors have addressed this problem recently: we refer the interested reader to the papers by Levine [4] and Deng and Levine [1] for a survey of the literature on this subject. In the proofs we follow the technique developed by Mitidieri and Pohozaev in [6, 7], which allows to prove the nonexistence of not necessarily positive solutions avoiding the use of any comparison principle through the choice of suitable text functions and careful capacitary estimates. We emphasize that in the present paper we do not assume any sign condition on the solutions, while we ask that the initial data have the following weak weighted positivity property:

$$\liminf_{R\to\infty}\int_{B_R} u_0|x|^{-\tau_1}\,dx > 0, \qquad \liminf_{R\to\infty}\int_{B_R} v_0|x|^{-\tau_2}\,dx > 0, \tag{1.3}$$

where $\tau_1 = \tau_2 = 0$ in case of system (1.2). Of course, (1.3) is in particular satisfied by positive initial data.

Throughout the paper by "nonexistence of weak solution" we mean "nonexistence of nontrivial weak solutions."

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