Editorial

Completely Monotonic and Related Functions: Their Applications

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Completely monotonic and related functions are important function classes in mathematical analysis. It was Bernstein [1] who in 1914 first introduced the notion of *completely monotonic function*. This year we celebrate its 100th anniversary. In 1921, Hausdorff [2] gave the notion of *completely monotonic sequence*, which is related to the notion of completely monotonic function. Widder [3] in 1931 introduced the notion of *minimal completely monotonic sequence*. The terminology: *logarithmically completely monotonic function*, was first used in [4] in 1988. In 1989, the authors [5] introduced the notion of *strongly completely monotonic function*. In 2009, the authors [6] introduced the notion of *strongly logarithmically completely monotonic function*. Also, the terminology: *almost strongly completely monotonic function*, was introduced in [6] in order to simplify the statement of the main results of the article [6]. In 2012, the terminology of *almost completely monotonic function* was defined and used in [7] in order to simplify the statement of the main results of the article [7]. Researchers did a lot of investigations, in both theory and applications, on these functions. More recently, for example, the authors [6] showed that a strongly logarithmically completely monotonic function must be almost strongly completely monotonic. Also, in [6], the following fact that a strongly logarithmically completely monotonic function cannot be strongly completely monotonic, or, equivalently, a strongly completely monotonic function cannot be strongly logarithmically completely monotonic was established. Figure 1 illustrates the inclusion relationships among these classes of functions on $(0, \infty)$. In these inclusion relationships, the fact that a logarithmically completely monotonic function must be completely monotonic was, in fact, first proved in [8] although the author did not use the terminology of *logarithmically completely monotonic function*; all others are from [6] or directly from their definitions. Also note that in [6] counter examples were presented to show that some function classes are not included in some other function classes or to show that the intersection sets of two function classes are not empty.

We invited investigators to contribute original research articles as well as comprehensive review articles to this special issue which will stimulate the continuing efforts to understand these functions. This special issue mainly focuses on the applications, in all fields, of completely monotonic and related functions. The topics included in this special issue are as follows:

(i) completely monotonic functions and their applications,

(ii) almost completely monotonic functions and their applications,

(iii) logarithmically completely monotonic functions and their applications,

(iv) strongly logarithmically completely monotonic functions and their applications,