

Editorial

Dynamic Systems and Related Algebra with Applications

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Dynamical systems have been widely used to describe the long-time behaviors in almost all areas of science and engineering. Using an algebraic approach to study dynamical systems has become more and more important in the dynamical systems theory and has drawn a great interest to many researchers recently.

Although dynamical systems theory and algebra seem to belong to two different areas of mathematics, they have a common generic structure. The study of the relationship between them has been active and very fruitful. The essence on this research is how to find different ways to associate algebra to dynamical systems in such a way that the algebra reflects the structure and the behavior of the solutions of the corresponding dynamical systems. As shown by the research, algebra can play a key role in the theory of dynamical systems. Topics in algebra such as similarity of matrices, eigenvalues, and (generalized) eigenspaces have been applied, recharacterized, and generalized in the dynamical systems theory. The most basic form of this interplay can be seen when a matrix A gives rise to a dynamical system. Matrices define nonlinear systems on smooth manifolds. The behavior of systems is closely related to matrices and their properties [1–6].

In recent years this correspondence between dynamical systems and algebra has evolved rapidly, especially in the area of using the invariant theory of algebra in the classification of the dynamical systems [7–10]. In the late nineteenth century, in order to unify and extend the methods of solving ordinary differential equations, a Norwegian mathematician, Lie, inspired by Galois and Abel in their treatment of algebraic equations, introduced the concept of continuous Lie group.

The works of Lie were systematic and all around including integral factor, homogeneous equation, order reduction of linear equations, and Laplace transformation of Euler equation. Many important properties of solutions of differential equations can be represented and described by the corresponding matrix A . If A is solvable, then the solution of the corresponding differential equation can be obtained through a linear differential equation. Automorphism keeps the equilibrium, periodic orbits, and the attraction region. Equilibrium of the differential equation can be obtained from the nilpotent element with index of 2 ($N^2 = 0$) of A , and idempotents ($E^2 = E$) give ray solutions. The origin is never asymptotically stable, and the existence of nonzero idempotents implies that the origin is actually unstable.

The description of characteristics of dynamical systems by using algebra theory, especially matrix space and matrix algebra theory, can be regarded as a natural generalization of classical Galois theory. The dynamical systems in physical, biological, and other natural sciences usually have some special algebraic properties, and they are often very interesting. An important example is the equivariant bifurcation theory, which in a natural way can be regarded as the application of Lie group in symmetric system.

Symmetric dynamics is generated from a kind of special systems with symmetric properties in the state space. It is widely used in engineering, cell, crystal structure, and so on. It is highly nontrivial to determine the symmetries, the stabilities, and branching patterns for solutions of equations equivariant under a compact Lie group G . From a mathematical point of view, symmetry refers to the system as an invariant property in a group action [11–14]. The invariance means that