

## Editorial

# Nonlinear Functional Analysis of Boundary Value Problems: Novel Theory, Methods, and Applications

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Received 24 February 2013; Accepted 24 February 2013

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Nonlinear boundary value problems often arise from scientific research, modelling of nonlinear phenomena, and optimal control of complex systems. In some cases, it is not possible to solve the underlying nonlinear boundary value problems in a closed form, and, for these cases, nonlinear functional analysis can play its role as a tool for obtaining qualitative information about the solution. The primary problems concerned in functional analysis are as follows: does a solution of the boundary value problem exist? If so, is it unique and stable? If the answers to these questions are affirmative, the problem is well posed and the analysis can proceed in a firm base to the next step—construct a method for finding approximate solutions and evaluate the quality of the approximation.

Over the last few decades, intensive research has been carried out worldwide to develop functional analysis theories and methods to tackle complex boundary value problems arising continually from scientific research and modelling of real world phenomena. The papers selected for this special issue represent a typical set of contributions in this field of research. Of course, the selected topics and papers are not an exhaustive representation of the recent development in the field. Nevertheless, they provide novel functional theories and methods as well as rich analysis for many complex boundary value problems with different application backgrounds and we have the pleasure of sharing these research results with the readers. Here, we would like to thank the authors and reviewers of the papers for their excellent

contributions. The efforts made by staff of the editorial office of the publishing corporation are also greatly acknowledged.

This special issue contains twenty-nine papers, covering functional analysis of various types of complex boundary value problems for different differential equations and boundary conditions. The particular focus is on fractional order differential equations and partial differential equations, and the boundary conditions include Riemann-Stieltjes integral boundary conditions and nonlocal boundary conditions.

The first set of papers, including nine papers, focus on the study of various complex fractional order boundary value problems.

- (i) In the paper titled “*Existence of positive solution for semipositone fractional differential equations involving Riemann-Stieltjes integral conditions*,” based on the fixed-point theorem in a cone, the existence of at least one positive solution is established for a class of semipositone fractional differential equations with Riemann-Stieltjes integral boundary condition.
- (ii) In the paper titled “*Multiple solutions for a class of fractional boundary value problems*,” the authors establish conditions for the existence of multiple solutions for a class of fractional boundary value problems involving left and right Riemann-Liouville fractional integrals of order  $0 \leq \beta < 1$  by the variational method.