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Multi-dimensional integral limit theorems for large deviations

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1. Introduction

The problem of large deviations in the central limit theorem was first treated by Khintchine [3] in a special case and later by Cramér [2] in a more general one-dimensional case. His results were slightly improved by Petrov [4], who studied the distribution of sums of independent but not necessarily identically distributed random variables. Richter has proved local central limit theorems in the one-dimensional case [5] and in the multi-dimensional case [6], when the distribution of the sum is either absolutely continuous or of lattice type. He has also stated theorems of integral type [7], but, as he pointed out, he was obliged to restrict himself to the above-mentioned special cases, mainly because the ordinary integral limit theorems were lacking.

Here I want to use the results obtained in [1] to generalize Richter's results in [7] and one of Cramér's results [2] to the multi-dimensional case. I shall only treat the case of a sum of independent and identically distributed random vectors (r.v.'s), the generalization to non-identically distributed r.v.'s being straightforward but somewhat cumbersome.

2. Statement of the problem

Let $X=(X_1, \ldots, X_k)$ be a r.v. in R_k, $k>1$, with the distribution function (d.f.) $F(x)$, $x=(x_1, \ldots, x_k)$, with zero mean and non-singular covariance matrix M. Furthermore let, for some $h_0>0$, the moment generating function (m.g.f.) of X,

$$R(t)=\int_{R_k} e^{(t,x)}dF(x), \quad (t,x)=\sum_{j=1}^{k} t_j x_j$$

exist for all $t=(t_1, \ldots, t_k)$ with $|t|=(\sum_{j=1}^{k} t_j^2)^{\frac{1}{2}}<h_0$

If $X^{(1)}, \ldots, X^{(n)}$ is a sequence of independent r.v.'s with the same d.f.'s as X, and $Y_n=(1/\sqrt{n})\sum_{\nu=1}^{n} X^{(\nu)}$, the problem is to estimate the probability $P(Y_n \in B)$, where B is a Borel set of a type specified in section 4. In Theorem 1, B is contained in a sphere with its center in the origin and of radius $R \leqslant \varepsilon_0\sqrt{n}$, and in Theorem 2, B is contained in the complement of such a sphere. In Theorems 3 and 4, I give applications to the d.f.'s of $|Y_n|$ and Y_n respectively.