ARKIV FÖR MATEMATIK Band 7 nr 9

1.66 14 1 Communicated 9 November 1966 by O. FROSTMAN and L. CARLESON

On the existence of the scattering operator

By EINAR LUNDQVIST

1. Introduction

Scattering, in its most simple form, can be described as a process during which two elementary particles collide. The particles are assumed to be infinitely separated in space both at the beginning and at the end of the process. The state at each instant is given by a function $\psi \in L^2(E_m)$. The particles interact with a certain force V, which decreases to zero as the distance between the particles tends to infinity. With each initial state (at the time $t=-\infty$) is uniquely associated a final state (at the time $t=\infty$) by means of the Schrödinger equation

$$i\frac{\partial}{\partial t}\psi(t) = H_1\psi(t), \tag{1}$$

where H_1 is the total Hamiltonian operator. H_1 is defined as a self-adjoint extension of the operator H_0+V, where H_0 represents the kinetic energy and V the potential energy. H_0 and V are self-adjoint. (1) describes the time development

$$\psi(s) = e^{-iH_1(s-t)}\psi(t) \tag{2}$$

for a state ψ.

One introduces a time-dependent representation of the Hilbert space $L^2(E_m)$ so that $e^{itH_0}\psi$ represents the function $\psi \in L^2(E_m)$. In this representation, "the interaction picture", the unitary operator $U(s,t)=e^{iH_0 s}\,e^{-iH_1(s-t)}\,e^{-iH_0 t}$ takes a state at the time t to the corresponding state at the time s. The operator U has the following properties

$$\left.\begin{aligned} U(s,t) &= U(s,t')\,U(t',t),\\ U^*(s,t) &= U(t,s). \end{aligned}\right\} \tag{3}$$

Under the assumption that $\lim_{t\to\pm\infty} U(0,t) = U(0,\pm\infty)$ exist in some sense,one defines the scattering operator S as

$$S = U^*(0,\infty)\,U(0,-\infty). \tag{4}$$

Then S transfers an initial state to the corresponding final state.

In the formal scattering theory one assumes that the wave operators $U(0,\pm\infty)$ exist and that S is unitary. A rigorous mathematical theory for scattering was