Communicated 10 November 1948 by T. CARLEMAN and F. CARLSON

Zeros of successive derivatives

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With 7 figures in the text

0. Introduction

We are going to study the distribution of the zeros of the successive derivatives of analytical functions. The main problem is to find the distribution of the limit points of these zeros. The concept of limit point will be used in the following sense. A point x is a limit point of the zeros of the derivatives, if and only if to every neighbourhood of x there is an infinity of derivatives which have zeros in this neighbourhood. Thus e. g. if the analytical function in question can be developed around the origin in a power series with gaps, then the origin is a limit point of the zeros of the derivatives.

0.1. The following notation will be used

$f(z)$	= the given analytical function.
$a_n(x)$	$= \dfrac{f^{(n)}(x)}{\lfloor n}$; $\because$ $f(z) = \sum_{n=0}^{\infty} a_n(x)(z-x)^n$.
$g_{nm}(x)$	$= \sqrt[n]{a_n(x)}$ where the index m indicates the branch of $\sqrt[n]{a_n}$. m is assumed to take one of the values $0; 1; \ldots (n-1)$.
ν	denotes a variable which takes the values $1, 2, 3 \ldots$
n_ν	denotes a monotonic sequence of natural numbers.
$C(\alpha; \beta)$	denotes *the circumference* of the circle with center α and radius β.
D	denotes a bounded simply connected open domain which does not contain any limit point of the zeros of the derivatives of f.
D_-	denotes a simply connected closed domain $\subset D$ (D_- is to be thought of as being very close to D).
$x \in D$	will often be used to express that x is not a limit point of zeros. When used with this meaning the relation is to be read thus: x is an element of some D. In an analogous way the relation $x \notin D$ will often mean that x is a limit point, and ought to be read: x is not an element of any D. This meaning of the symbols $x \in D$ and $x \notin D$ is used when D is not specified.
$R(x)$	= distance from x to nearest singular point of $f(z)$.