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On linear estimates defined by a continuous weight function

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With 1 figure in the text

1. Introduction

Let X be a random variable having the cumulative distribution function (cdf) $F\left(\frac{x-\mu}{\sigma}\right)$, where $F(y)$ is a known cdf and where μ and σ are unknown parameters. From n independent observations of X, we obtain by rearranging them in order of magnitude the order statistics $x_{(1)} \leq x_{(2)} \leq \cdots \leq x_{(n)}$. A linear estimate of μ or σ is a systematic statistic

$$\theta^* = \sum_{\nu=1}^{n} h_\nu^{(n)} x_{(\nu)}.$$

Assuming the means and the covariances of the standardized order statistics $y_{(\nu)} = \frac{x_{(\nu)}-\mu}{\sigma}$ to be known, the constants $h_\nu^{(n)}$ may be determined so as to make θ^* unbiased and of minimum variance. (LLOYD [4].) In a few special cases explicit solutions of the constants $h_\nu^{(n)}$ are obtained (DOWNTON [3], SARHAN [6]). In the general case much numerical work is needed.

In this note, we put $h_\nu^{(n)} = \frac{1}{n} h\left(\frac{\nu}{n+1}\right)$, where $h(u)$ is a continuous function, depending on $F(y)$ only, and we shall prove that $h(u)$ may often be chosen so as to make the corresponding estimate $\theta^*(h)$ consistent and asymptotically efficient.

In section 2 we study the properties of the statistic $\theta^*(h)$ for an unspecified function $h(u)$. In the next section we solve a special minimum problem, and the solution is applied to the determination of the best weight function in section 4, where we also make the transformations necessary for practical use. The last sections deal with the asymptotic efficiency of the estimates obtained and some applications.

2. Asymptotic expressions for the mean and the variance of a certain type of linear systematic statistics

Let $y_{(1)} < y_{(2)} < \cdots < y_{(n)}$ be the order statistics from a sample of n independent observations of the random variable Y. We assume that Y has the cdf $F(y)$, a continuous probability density function (pdf) $f(y)$ and a finite second moment.