Ark. Mat., **35** (1997), 277–297

Uniform growth of analytic curves away from real points

Rüdiger W. Braun, Reinhold Meise and B. Alan Taylor

1. Introduction

Let V be a one-dimensional analytic curve in $\{(t,s)\in\mathbf{C}^2:|s|<1\}$ such that the projection map $\pi(t,s)=s$ onto the second coordinate is proper on V. Then there is an integer ν such that, except over a discrete subset of $|s|<1$,

$$V=\{(t_j(s),s):1\leq j\leq\nu,\ |s|<1\}; \tag{1}$$

that is, V is the zero set of the monic pseudopolynomial with coefficients analytic for $|s|<1$,

$$F(t,s)=\prod_{j=1}^{\nu}(t-t_j(s))=t^\nu+\sum_{j=0}^{\nu-1}a_j(s)t^j.$$

Each of the branches $t_j(s)$ then has a Puiseux series expansion about $s=0$ of the form

$$t_j(s)=t_j(0)+\sum_{k=1}^{\infty}d_{j,k}s^{k/N}$$

for some integer $N\geq1$. Suppose that there is a constant $C>0$ and a rational number $r=p/q$, $0<p/q\leq1$, such that

$$|\operatorname{Im}t_j(s)|\leq C|s|^{p/q},\quad s\text{ real},\ |s|<1.$$

This condition implies that $t_j(0)$ is real and that the first fractional power k/N that has a nonzero coefficient $d_{j,k}$ in the series must satisfy $k/N\geq p/q$. Hence,

$$|t_j(s)-t_j(0)|\leq C'|s|^{p/q},\quad |s|<1,\ 1\leq j\leq\nu \tag{2}$$

for some constant C' which, a priori, depends on the curve V. We are going to prove that the constant can be chosen independent of the curve V, in the following sense.