Ark. Mat., **33** (1995), 323–341

Fredholm property of partial differential operators of irregular singular type

Masatake Miyake and Masafumi Yoshino(1)

1. Introduction

In 1974 Kashiwara–Kawai–Sjöstrand showed the sufficient condition for the convergence of all formal power series solutions of the following linear partial differential equations of regular singular type

$$\mathcal{L}(x,D)u(x)\equiv\sum_{|\alpha|=|\beta|\le m} a_{\alpha\beta}D^{\beta}(x^{\alpha}u(x))=f(x), \tag{1.1}$$

where m is a positive integer and $a_{\alpha\beta}$'s are complex constants. Here we use the standard notations of multi-indices, $D^{\beta}=(\partial/\partial x_1)^{\beta_1}\dots(\partial/\partial x_n)^{\beta_n}$, $|\alpha|=\alpha_1+\dots+\alpha_n$ and $x^{\alpha}=x_1^{\alpha_1}\dots x_n^{\alpha_n}$. They proved the following result.

Theorem 1.1. (cf. [4]) *Suppose that the following condition*

$$\sum_{|\alpha|=|\beta|=m} a_{\alpha\beta}z^{\alpha}\bar{z}^{\beta}\neq 0, \tag{1.2}$$

is satisfied for any $z\in\mathbf{C}^n\setminus\{0\}$, *where* $z^{\alpha}=z_1^{\alpha_1}\dots z_n^{\alpha_n}$ *and* $\bar{z}^{\beta}=\bar{z}_1^{\beta_1}\dots\bar{z}_n^{\beta_n}$. *Then, for any* $f(x)$ *analytic at the origin all formal power series solutions* $u(x)$ *of the equation* (1.1) *converge in some neighborhood of the origin.*

They proved results for somewhat more general operators than (1.1) admitting perturbations.

Inspired from this theorem we shall study in this paper the Fredholm property of regular and irregular singular type operators including (1.1) in (formal) Gevrey spaces in a neighborhood of the origin of $\mathbf{C}^2$. We introduce a Toeplitz symbol in

(1) Supported by Chuo University Special Research Program (1994)