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The propagation of polarization for systems of transversal type

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1. Introduction

In this paper we study the propagation and distribution of polarization sets for solutions to systems having characteristics of transversal involutive self-intersection. Thus, we assume that the characteristic set is micro-locally a union of two non-radial hypersurfaces, which have transversal involutive intersection at the double characteristics. We also assume that the principal symbol vanishes of first order on the two-dimensional kernel at the intersection. These types of systems we call systems of transversal type. The propagation of singularities for the corresponding scalar wave operator was considered in [11].

We shall consider the propagation of $H_{(s)}$ polarization sets of the solutions. This polarization set indicates those components of the distribution, which are not in $H_{(s)}$. Outside the intersection of the characteristics, the polarizations for solutions propagate along Hamilton orbits, which are unique liftings of the bicharacteristics. The limits of polarizations from outside the double characteristic set, are called real polarizations, the others are called complex polarizations. It follows from the conditions that there are only two linearly independent real polarizations over the double characteristic set. The real polarizations are foliated by limits of Hamilton orbits, which we call limit Hamilton orbits. The results on the propagation of polarization depend on whether the polarization is contained in (limit) Hamilton orbits or not. When it is, we can define an invariant, called the trace of the orbits (see Definition 5.3). If this trace satisfies a second order transport equation along the bicharacteristics, we obtain propagation of polarization according to Theorem 6.2. When the polarization is either complex, or real and transversal to limit Hamilton orbits, we prove propagation of polarization in Theorems 6.3 and 6.4, respectively.

When we have a polarization condition, i.e., one component of the solution is in $H_{(s)}$, then the singularities of the solutions must either be contained in limit