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Homogeneous Fourier multipliers of Marcinkiewicz type

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1. Introduction

Let $m\in L^\infty(\mathbf{R}^2)$ be homogeneous of degree zero. Then m is almost everywhere determined by $h_\pm(\xi_1)=m(\xi_1,\pm 1)$. For $k\in\mathbf{Z}$ let $I_k=[2^{-k-1},2^{-k}]\cup[-2^{-k},-2^{-k-1}]$ and let h_+ and h_- satisfy the condition

$$\sup_{k\in\mathbf{Z}}\left(\int_{I_k}|sh'_\pm(s)|^r\frac{ds}{s}\right)^{1/r}<\infty. \tag{1.1}$$

Rubio de Francia posed the question whether a condition like (1.1) is sufficient to prove that m is a Fourier multiplier of $L^p(\mathbf{R}^2)$, $1<p<\infty$. An application of the Marcinkiewicz multiplier theorem with L^2-Sobolev hypotheses (cf. (1.3) and (1.5) below) and interpolation arguments already show that the answer is yes, provided $r>2$. Recently, Duoandikoetxea and Moyua [15] have shown that the same conclusion can be reached if $r=2$. On the other hand, since characteristic functions of halfspaces are Fourier multipliers of L^p, $1<p<\infty$, a simple averaging argument shows that the condition $h'\in L^1$ implies L^p-boundedness for $1<p<\infty$. Our first theorem shows that the weaker assumption (1.1) with $r=1$ implies boundedness in $L^p(\mathbf{R}^2)$, for $1<p<\infty$.

Theorem 1.1. *Suppose that h_+ and h_- satisfy the hypotheses of the Marcinkiewicz multiplier theorem on the real line, that is*

$$\sup_{k\in\mathbf{Z}}\int_{I_k}|dh_\pm(s)|\le A \tag{1.2}$$

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