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Expansions for Eisenstein integrals on semisimple symmetric spaces

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1. Introduction

Let G/H be a semisimple symmetric space. Related to the (minimal) principal series for G/H there is a series of Eisenstein integrals on G/H. These are K-finite joint eigenfunctions for the G-invariant differential operators on G/H. Here K is a maximal compact subgroup of G. The Eisenstein integrals are generalizations of the elementary spherical functions for a Riemannian symmetric space (and more generally of the generalized spherical functions in [9, §III.2]), and of Harish-Chandra's Eisenstein integrals associated to a minimal parabolic subgroup of a semisimple Lie group.

In this paper we develop a theory of asymptotic (in fact, converging) expansions towards infinity for the Eisenstein integrals. The theory generalizes Harish-Chandra's theory (see [8, Thm. IV.5.5], and [13, Thm. 9.1.5.1]) in the two cases mentioned above (see also [9, Thm. III.2.7]). The main results are Theorems 9.1 and 11.1. The first of these states the convergence on an open Weyl chamber of the series expansion whose coefficients are derived recursively from the differential equations satisfied by the Eisenstein integrals. The sum Φ_λ of the series is an eigenfunction which behaves regularly at infinity but in general is singular at the walls of the chamber. The basic estimates which ensure the convergence of the series also provide an estimate for Φ_λ, which is a generalization of Gangolli's estimates ([7]) in the Riemannian case. As in Gangolli's case, our estimates are derived by a modification of the Φ_λ with the square root of a certain Jacobian function.

The second main result expresses the Eisenstein integral as a linear combination of the Φ_λ; the coefficients are the c-functions (defined in previous work by one of us) related to the Eisenstein integrals.

The results of this paper are used for the Plancherel and Paley–Wiener type results obtained in [5] for the Fourier transform corresponding to the minimal prin-