APPLICATIONS OF ALMOST PERIODIC COMPACTIFICATIONS

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1. Introduction

The theory of almost periodic functions on groups can be completely reduced to the study of continuous functions on compact topological groups by the introduction of the almost periodic compactification (see [1] or [14]). There are many possible constructions of the compactification; one of these is the following. If A is the space of almost periodic functions on a group G, and $\mathcal{B}(A)$ is the space of bounded linear operators on A, the compactification can be taken to be the closure in $\mathcal{B}(A)$, in the strong operator topology, of the group of right translates of A by elements of G.

This type of construction is of a very general nature and is peculiar neither to the strong operator topology nor to groups of operators. The purpose of this paper is to exhibit some extensions of this construction and applications of the resulting compactifications.

For example, in the above construction, if A is taken to be the space of weakly almost periodic functions (in the sense of [7]) on G, the closure in the weak operator topology of the right translates of G on A yields a compactification that is in general no longer a group but is a compact semigroup in which multiplication is separately continuous. This allows us to reduce, in a manner completely analogous to the almost periodic case, the theory of weakly almost periodic functions on groups to the study of continuous functions on such compact semigroups. As a consequence of the ideal structure for these semigroups (cf. Section 2), we indicate in Section 5 how the Eberlein theory of weakly almost periodic functions on locally compact abelian groups can be extended to a large class of groups and semigroups. In particular we show when and how a mean, and thus the possibility of Fourier analysis, arises for weakly almost periodic functions.