ON EMBEDDINGS OF SPHERES

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Introduction

If we embed an $(n-1)$-sphere in an n-sphere, the complement consists of two components. Our problem is to describe the components more exactly.

For $n=2$, there is a classical theorem of Schöenflies which says that an arbitrary simple closed curve in the two-dimensional sphere S^2 separates S^2 into two components whose closures are both topologically equivalent to a disk. The Riemann mapping theorem yields, moreover, a conformal equivalence between the interior of a simple closed curve and the open disk.

The reasonable conjecture to make would be that some analogous result holds for all dimensions; more precisely, that the complementary components of an $(n-1)$-sphere embedded in n-space are topologically equivalent to n-cells.

A classical counter-example (in dimension $n=3$) to this unrestricted analogue of the two-dimensional Schöenflies theorem is a wild embedding of S^2 in S^3 known as the Alexander Horned Sphere [1]. One of the complementary components of this embedding is not homeomorphic with the n-cell, and, in fact, not simply connected.

One's intuition shrugs at this counter-example, attributes its existence to the 'pathology of the non-differentiable', or whatever, and persists in believing the statement true—at least for nice imbeddings. In particular, the Alexander Horned Sphere embedding can be made neither differentiable nor polyhedral.

Under the assumption that the two-sphere S^2 is embedded polyhedrally in S^3, Alexander [1], and later, Moise [4], proved that the closures of the complementary components of S^2 were topological 3-cells.

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(1) A research announcement has already appeared in *Bull. Amer. Math. Soc.* 65, 1959.