On the distribution of Sidon series

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1. Introduction

Suppose that G is a compact abelian group with dual group Γ. Denote the normalized Haar measure on G by μ. Let $\mathcal{C}(G)$ be the Banach space of continuous complex-valued functions on G. If $S \subset \Gamma$, a function $f \in L^1(G)$ is called *S-spectral* whenever $\hat{f}$ is supported in S, where here and throughout the paper $\widehat{\ }$ denotes taking the Fourier transform. The collection of S-spectral functions that belong to a class of functions $\mathcal{W}$ will be denoted by $\mathcal{W}_S$.

Definition 1.1. A subset E of Γ is called a Sidon set if there is a constant $c>0$, depending only on E, such that

$$\sum_{\gamma\in\Gamma} |\hat{f}(\gamma)| \le c\,\|f\|_\infty \tag{1}$$

for every $f \in \mathcal{C}_E(G)$. The smallest constant c such that (1) holds is denoted by $S(E)$ and is called the constant of sidonicity of E, or the Sidon constant of E.

If $E=\{\gamma_j\}\subset\Gamma$ is a Sidon set and $\{a_j\}$ is a sequence in a Banach space B, then the formal series $\sum a_j\gamma_j$ will be referred to as a *B-valued Sidon series.* The norm on a given Banach space B will be denoted by $\|\cdot\|$, or, sometimes, by $\|\cdot\|_B$.

It is well-known that Sidon series share many common properties with Rademacher series. The following theorem of Pisier illustrates this fact and will serve as a crucial tool in our proofs.

Theorem 1.2 [Pi1, Théorème 2.1]. *Suppose that $E=\{\gamma_n\}\subset\Gamma$ is a Sidon set, that B is a Banach space, and that $a_1, \ldots, a_N \in B$. There is a constant c_1, depending*

([1]) Supported by NSF grant DMS 9102044
([2]) Supported by NSF grant DMS 9001796