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Hyperbolicity of localizations

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1. Introduction

Let $P(x,D)$ be a differential operator of order m in an open set $\Omega\subset\mathbf{R}^{n+1}$ with coordinates $x=(x_0,x')=(x_0,x_1,...,x_n)$, hence a sum of differential polynomials $P_j(x,D)$ of order j $(j\leq m)$ with symbols $P_j(x,\xi)$. In [7] Ivrii–Petkov has proved a necessary condition for the Cauchy problem of $P(x,D)$ to be correctly posed which asserts that $P_{m-j}(z)$ must vanish of order $r-2j$ at z if $P_m(z)$ vanishes of order r at z with $z=(x,\xi)\in T^*\Omega\backslash 0$. This enables us to define the localization $P_{z_0}(z)$ at a multiple characteristic z_0 (of $P_m(z)$) following Helffer [4] which is a polynomial on $T_{z_0}(T^*\Omega)$.

In this note we show that $P_{z_0}(z)$ is hyperbolic, that is verifies Gårding's condition if the Cauchy problem for $P(x,D)$ is correctly posed. The proof is based on the arguments of Svensson [9].

Since $P_{z_0}(z)$ is hyperbolic one can define the localizations $P_{(z_0,z_1,...,z_s)}(z)$ successively as the localization of $P_{(z_0,z_1,...,z_{s-1})}(z)$ at z_s which are hyperbolic polynomials on $T_{z_0}(T^*\Omega)\cong...\cong T_{z_s}(T^*\Omega)$ (see Hörmander [6, II] and Atiyah–Bott–Gårding [1]). It may happen that the lineality $\Lambda_{(z_0,z_1,...,z_s)}(P_m)$ of $P_{m(z_0,z_1,...,z_s)}(z)$ is an involutive subspace with respect to the canonical symplectic structure on $T_{z_0}(T^*\Omega)$. In this case we prove that for the Cauchy problem to be correctly posed it is necessary that

$$P_{(z_0,z_1,...,z_s)}(z)=P_{m(z_0,z_1,...,z_s)}(z).$$

This argument was also used in Bernardi–Bove–Nishitani [2] with $s=1$.

2. The localization is hyperbolic

We denote by $L^{m,r}_{z_0}$ the set of pseudodifferential operators P near z_0 with