THE METHOD OF SUCCESSIVE APPROXIMATIONS FOR FUNCTIONAL EQUATIONS.

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In the functional analysis abstract linear spaces are considered which may have for their elements mathematical objects of a various nature: numbers, sequences of numbers, functions etc. Therefore theorems established for such abstract spaces usually can be applied to very different branches of mathematical analysis. Thus the general theory of functional equations, i. e. of such equations where the unknown quantities are elements of a linear space, comprises the theories of differential, integral and some other equations considered in analysis as well as the theory of finite and infinite systems of algebraic equations. One of the most important methods for establishing the existence of solutions and for the investigations of these solutions is the method of successive approximations. We shall give here the general theory of this method for linear and non-linear functional equations in a very wide class of spaces viz. the spaces normed with the elements of a semi-ordered space. This class comprises inter alia Banach's spaces and semi-ordered spaces.[1] The theory of this method will be based on the principle of majorants. We shall give also some applications of the general theory to the systems of algebraic equations and to the differential and integral equations.

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[1] For the case of semi-ordered spaces some theorems on these functional equations have been published already by the author of the present paper. See List of Literature, vide KANTOROVITCH, I.