SOLUTION OF THE PLATEAU PROBLEM FOR m-DIMENSIONAL SURFACES OF VARYING TOPOLOGICAL TYPE

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Introduction

The Plateau Problem consists in showing that the greatest lower bound of the areas of surfaces with a given boundary is attained. This depends primarily on the meaning we attach to the word surface. In the classical conception we start with a two-dimensional manifold R with boundary C and a set A homeomorphic to C; we then say that S is a surface of class $\mathcal{G}_R$ with boundary A if there is a continuous mapping of R onto S which maps C onto A $(1-1)$ and bicontinuously. In this sense the problem was very elegantly solved by Jesse Douglas in the 1930's [5].

But this solution leaves the question incomplete in a number of important respects. In the first place the problem so posed deals only with a class of surfaces which are all of the same topological type and for each topological type we have a separate theorem. Now if we are prepared to consider as a surface any set S which is a mapping of a manifold R whose boundary is homeomorphic to A, then surely we should be interested in the class of all such S where R is allowed to vary. When dealing with the problem in this light it seems intuitive that a minimum will be attained provided we admit as surfaces sets which, while they are manifolds at points away from the boundary, will, when the boundary is complicated, have infinitely many loops and infinite connectivity near the boundary. However this involves a comparison between surfaces which are not mappings of a common base space, and the classical methods are inherently very ill adapted for this. In the second place while it is intuitive that any set which is a surface of minimum area in some sense will be locally well-behaved, this is a result which it would be nice to prove and this cannot be done if we only consider locally well-behaved surfaces in the first place. In other