A STRUCTURE THEORY FOR A CLASS OF LATTICE-ORDERED RINGS

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The foundations of a systematic general theory of lattice-ordered rings were laid by Birkhoff and Pierce in [3]. They introduced, as an object for further study, the class of f-rings. This special class of lattice-ordered rings displays a rich structure: it can be characterized as the class of all subdirect unions of ordered rings. Birkhoff and Pierce obtained many properties of f-rings, basing their structure theory on the l-radical of an f-ring. In [20], Pierce obtained an important decomposition theorem for f-rings with zero l-radical. This paper continues the study of the structure of f-rings.

In Chapter I, we present the necessary background material and obtain a characterization of the l-radical of an f-ring that yields a new proof of the decomposition theorem of Pierce.

In Chapter II, we present a structure theory for f-rings based on an f-ring analogue of the Jacobson radical for abstract rings. In Section 1, the J-radical of an f-ring A is defined to be the intersection of the maximal modular right l-ideals of A. In Section 2, the J-radical is characterized in terms of the notion of l-quasi-regularity, and this characterization is used to obtain certain properties of the J-radical.

In Section 3, we consider a representation theory for f-rings. We show that every f-ring that has a faithful, irreducible, l-representation is a totally ordered ring with identity that contains no non-zero proper one-sided l-ideals. In Section 4, the notions of l-primitive f-ring

(1) This paper is based on the author's doctoral dissertation, written under the supervision of Prof. Melvin Henriksen. The author wishes to thank Prof. Henriksen for his advice and encouragement. Financial support for this research was provided by the National Science Foundation, U.S.A., under contract No. NSF G-3021. and by the Office of Naval Research, U. S. Navy, under contract No. Nonr 1100(12).