ASYMPTOTIC BEHAVIOR OF SOLUTIONS OF PARABOLIC EQUATIONS OF ANY ORDER

BY

AVNER FRIEDMAN

Minneapolis, U.S.A.[1]

Introduction

For some parabolic differential equations it is known that any solution in a cylindrical domain with axis $t>0$, tends to a limit as $t\to\infty$ provided the boundary values and the coefficients of the equation tend to a limit as $t\to\infty$. Furthermore, the limit of the solution is known to be the solution of the limit equation. For second order parabolic equations, this has been proved by the author [5] for the first mixed boundary value problem, that is, when the solution u is prescribed on the lateral boundary of the cylinder. Extension to equations with a nonhomogeneous term which is "slightly" nonlinear in u, is also given in [5]. In [6] it was proved that if both the coefficients of the parabolic equation and the boundary values admit an asymptotic expansion in t^{-1} $(t\to\infty)$, then the same is true of the solution. Asymptotic convergence for solutions of second order parabolic equations satisfying a nonlinear boundary condition (generalized Newton's law of cooling) was established by the author in [7].

The present paper consists of two parts. In Part I we consider second order parabolic equations and establish the asymptotic behavior of solutions, both for the first and the second (and even more general) mixed boundary value problems. The nonhomogeneous term is a nonlinear perturbation. The domains are "almost cylindrical," i.e., the cross sections $t=$ const. tend to a limit as $t\to\infty$. For the first mixed boundary value problem, the present treatment is not only an improvement of the analogous results of [5], but it is also a much more simplified treatment. Thus for instance, we do not make here any use of existence theorems for parabolic equations. We

(1) Part I was prepared under Contract Nonr-222 (62) (NR 041 214) with the University of California in Berkeley. Part II was prepared under Contract Nonr-710 (16) (NR 044 004) between the Office of Naval Research and the University of Minnesota.