ON FOURIER TRANSFORMS OF MEASURES WITH COMPACT SUPPORT

BY

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Introduction

This paper will deal with the set $\mathfrak{M}$ of measures with compact support on the real line. To each positive number a we associate the set $\mathfrak{M}_a$ consisting of measures with support contained in $[-a, a]$. $\hat{\mathfrak{M}}$ and $\hat{\mathfrak{M}}_a$ will denote the sets of Fourier transforms $\hat{\mu}$ for μ belonging to $\mathfrak{M}$ and $\mathfrak{M}_a$ respectively. By reason of convenience the identically vanishing measure shall not be included in $\mathfrak{M}$ or $\mathfrak{M}_a$.

Our main objective is to decide if for each $a>0$ there exists $\mu \in \mathfrak{M}_a$ which tend to 0 in a prescribed sense as $x \to \pm\infty$. Since each $\hat{\mu}(x) \in \hat{\mathfrak{M}}$ is the restriction to the real axis of an entire function of exponential type $\leqslant a$, bounded for real x, we know by a classical theorem that

$$J(\log^-|\hat{\mu}|) = \int_{-\infty}^{\infty} \frac{\log^-|\hat{\mu}(x)|}{1+x^2}\,dx > -\infty. \tag{0.0}$$

This property is therefore a necessary condition.

Let $w(x) \geqslant 1$ be a measurable function on the real line and let L^p_w $(1 \leqslant p \leqslant \infty)$ be the space of measurable functions $f(x)$ with norm

$$\|f\| = \left\{\int_{-\infty}^{\infty} |f(x)|^p\, w(x)^p\, dx\right\}^{1/p}.$$

The following problem will be considered. Determine for a given p the set W_p of all weight functions $w(x) \geqslant 1$ subject to these two conditions:

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