

THE MICROBUNDLE REPRESENTATION THEOREM

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The present paper contains a generalization of the Kister–Mazur theorem which says that any microbundle over a finite dimensional simplicial complex contains a (up to bundle isomorphism) unique fibre bundle. Precisely, we prove this theorem (or a relativized version of it) for microbundles over arbitrary topological spaces, provided the microbundle admits a trivializing partition of unity on the base. In particular the theorem applies to any microbundle over a paracompact space. At present this is a work that aims at generality and completeness rather than applicability, since so far the Kister–Mazur result covers most of the interesting cases. From a purely esthetical point of view, however, the latter has certain defects. The natural objects to study among the microbundles over a simplicial complex are the piecewise linear microbundles. For such one should of course expect sharper results. Recently Hirsch, Mazur and others have shown that a piecewise linear microbundle contains subcomplexes which are piecewise linear bundles and that any two such are piecewise linearly isomorphic [5]. On the other hand, in the category of topological microbundles it seems unnatural to put any restrictions at all on the base space.

The condition about the existence of a trivializing partition of unity has already been introduced on bundles by Dold, who calls such bundles *numerable*, cf. [3]. Any (micro-) bundle over a normal base space covered by a locally finite family of trivializing open sets is numerable. Products, sums and “pull-back’s” of numerable (micro-)bundles are numerable. Dold also shows that the numerable bundles have the good properties shared by bundles over paracompact spaces. In view of his work it almost seems desirable to redefine (micro-)bundles as numerable (micro-)bundles. In any case it has been convenient to do so here. By definition (micro-)bundles in this paper are always numerable.

Besides the techniques purified in [3], an inductive process of Mazur for extending homeomorphisms on open sets in $\mathbb{R}^q$ plays a fundamental role in the sequel.

A preliminary report on this work has already appeared in [6]. The present paper