NECESSARY DENSITY CONDITIONS FOR SAMPLING AND INTERPOLATION OF CERTAIN ENTIRE FUNCTIONS

BY

H. J. LANDAU

Bell Telephone Laboratories, Incorporated, Murray Hill, New Jersey, U.S.A.

Introduction

In a series of seminar lectures given in 1959–60 at the Institute for Advanced Study in Princeton, Professor Arne Beurling posed and discussed the following two problems, in Euclidean spaces:

A. *Balayage.* Let G be a locally compact abelian group, Λ a closed subset of G, and S a given collection of characters. Let $M(G)$ and $M(\Lambda)$ denote the sets of all finite Radon measures having support in G and Λ, respectively. Balayage was said to be possible for S and Λ if corresponding to every $\alpha \in M(G)$ there exists $\beta \in M(\Lambda)$ such that

$$\int \varphi \, d\alpha = \int \varphi \, d\beta, \quad \text{for all} \quad \varphi \in S.$$

Choice of this term was prompted by analogy with its original usage, in which S was a set of potential-theoretic kernels.

The set S, viewed as a subset of the dual group of G, was restricted from the outset to be compact, and to satisfy the regularity conditions (α) and (β) below.

(α) For each $s_0 \in S$ and each neighborhood ω of s_0, there exists a positive Radon measure having support in $\omega \cap S$, with Fourier transform approaching zero at infinity (i.e., outside compact subsets of G).

Let $C(G)$ be the space of bounded continuous functions on G with the uniform norm. Let the weak closure of a set $P \subset C(G)$ consist of those functions of $C(G)$ which are annihilated by every measure in $M(G)$ annihilating P. Let the spectral set Σ_φ of $\varphi \in C(G)$ consist of the characters contained in the weak closure of the set of all translates of φ, and let $C(G, S)$ denote the collection of all $\varphi \in C(G)$ with $\Sigma_\varphi \subset S$.