ON THE DISTRIBUTION OF VALUES OF MEROMORPHIC FUNCTIONS OF BOUNDED CHARACTERISTIC

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Introduction

1. Let $w = f(z)$ be a non-constant meromorphic function in the unit circle $|z| < 1$. Using the standard notation[1] we write $(a \neq f(0))$

$$N(r, a) = N\left(r, \frac{1}{f-a}\right) = \int_0^r \frac{n(r, a)}{r} \, d r,$$

where $n(r, a)$ denotes the number of the roots of the equation $f(z) = a$ in the disk $|z| \leq r$, multiple roots being counted with their order of multiplicity. For $a \to f(0)$ the above integral tends to the limit $+\infty$. By the customary definition of $N(r, a)$, this logarithmic singularity at $a = f(0)$ is removed, but in this paper we prefer permitting the existence of the singularity. For $\lim_{r \to 1} N(r, a)$ we write $N(1, a)$.

With the help of $N(r, a)$ the characteristic function $T(r)$ of $f(z)$ can be defined as the mean-value (Shimizu-Ahlfors's theorem)

$$T(r) = \int N(r, a) \, d\mu,$$

where the integral is extended over the whole plane and $d\mu$ denotes the spherical element of area divided by π, i.e.,

$$d\mu = \frac{|a| \, d|a| \, d \arg a}{\pi (1 + |a|^2)^2}.$$

According as $T(r)$ is bounded or not, the functions $f(z)$ meromorphic in $|z| < 1$ fall into two essentially different classes. If $f(z)$ is of bounded characteristic, then

[1] For the general theory of single-valued meromorphic functions we refer to NEVANLINNA [7].