SOLUTIONS OF DIFFERENTIAL EQUATIONS AS ANALYTIC FUNCTIONALS OF THE COEFFICIENT FUNCTIONS

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Introduction

In a series of papers during the last several years, ([1], [2], and references therein), one of us has developed a theory of the solutions of linear differential equations as analytic functionals of the coefficient functions. In the present paper, we consider a more general situation in which the differential equation is not restricted to be linear and use different methods. Even in the linear case, the results are a little different.

The method is to establish an implicit function theorem for analytic functions on one complex Banach space to another, and then apply this theorem to the differential equation.

Implicit functional equations in abstract spaces have been studied by various authors[1], and from various points of view. Since we restrict ourselves to the analytic case, it seemed appropriate to develop a theorem by generalizing the classical method of series expansions and dominating functions. A result similar to our theorem of Section 1 was given without proof by Michal and Clifford [3].

In the second section the implicit function theorem is used to study the solution of the differential equation $dy/d\tau = f(\tau, y)$ as a functional of the function f. Here τ is a real variable while y may range over a subset of a complex Banach space.

In particular the theory will include systems of ordinary differential equations and certain types of partial differential equations.

1. Implicit Functions

In the present section we shall make use of the abstract differential calculus and of the theory of analytic functions in complex Banach spaces.[2] In particular

[1] See, e. g. HILDEBRANDT and GRAVES [11].

[2] For a summary of these theories in complex Banach spaces see HILLE [6], chap. 4. For real as well as complex Banach spaces see MICHAL [1, 8]. MICHAL and MARTIN [9], MARTIN [10].