Acta Math., 194 (2005), 133–153

Distinguished varieties

by

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0. Introduction

In this paper, we shall be looking at a special class of bordered (algebraic) varieties that are contained in the bidisk $\mathbf{D}^2$ in $\mathbf{C}^2$.

Definition 0.1. A non-empty set V in $\mathbf{C}^2$ is a *distinguished variety* if there is a polynomial p in $\mathbf{C}[z,w]$ such that

$$V = \{(z,w)\in \mathbf{D}^2 : p(z,w)=0\}$$

and such that

$$\overline{V}\cap\partial(\mathbf{D}^2) = \overline{V}\cap(\partial\mathbf{D})^2. \tag{0.2}$$

Condition (0.2) means that the variety exits the bidisk through the distinguished boundary of the bidisk, the torus. We shall use ∂V to denote the set given by (0.2): topologically, it is the boundary of V within Z_p, the zero set of p, rather than in all of $\mathbf{C}^2$. We shall always assume that p is chosen to be minimal, i.e. so that no irreducible component of Z_p is disjoint from $\mathbf{D}^2$ and so that p has no repeated irreducible factors. Why should one single out distinguished varieties from other bordered varieties?

One of the most important results in operator theory is T. Andô's inequality [7] (see also [12] and [24]). This says that if T_1 and T_2 are commuting operators, and both of them are of norm 1 or less, then for any polynomial p in two variables, the inequality

$$\|p(T_1,T_2)\| \leqslant \|p\|_{\mathbf{D}^2} \tag{0.3}$$

holds. Andô's inequality is essentially equivalent to the commutant lifting theorem of B. Sz.-Nagy and C. Foiaş [23]—see, e.g., [20] for a discussion of this.

The first author was partially supported by the National Science Foundation. The second author was partially supported by National Science Foundation Grant DMS 0070639.