

The arithmetic and geometry of some hyperbolic three manifolds

by

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1. Introduction

In section 304 of his “Disquisitiones Arithmeticae” Gauss observed numerically that the asymptotic averages of class numbers of indefinite binary quadratic forms (over \mathbf{Z}) when ordered by their discriminants are rather erratic. Today the behavior of the class number for large discriminant still remains a major unsolved problem. In [17] we showed that if we form the averages of these class numbers when ordered by the sizes of the corresponding fundamental units (or regulators) then there is an asymptotic law. The main result of this paper is to derive similar such asymptotic expressions for averages of binary quadratic forms over the integers of an imaginary quadratic number field. As will be seen there are some interesting differences. We believe that our results have an appropriate extension to an arbitrary number field.⁽²⁾ To develop these results we will need to analyze in some detail various aspects of the geometry of certain hyperbolic three manifolds. Along the way various auxiliary theorems, which are of interest in their own right, concerning these manifolds, will be proven. These are stated in the following outline of the various sections.

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⁽²⁾ For the appropriate explicit trace formulas for totally real number fields see Efrat [4].