A geometric proof of Mostow's rigidity theorem for groups of divergence type

by

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1. Introduction

The nature of the "boundary map" of a geometric isomorphism between discrete Möbius groups Γ, Γ' acting on upper half n-space $\mathbf{U}^n$, has been studied extensively under various hypotheses on Γ. Without undue elaboration at this point, it comes down to a homeomorphism g of $\mathbf{R}^{n-1}=\partial\mathbf{U}^n$, with the property that for every $A\in\Gamma$, the composition $g\cdot A\cdot g^{-1}$ belongs to Γ' (as Γ' acts on $\mathbf{R}^{n-1}=\partial\mathbf{U}^n$).

The most dramatic result [8] is a special case of what has become known as Mostow's rigidity theorem, and states that if Γ is of finite covolume, and if $n\geqslant 3$, then g is conformal. Ahlfors gave a shortened proof in [3]. Mostow later [9] extended his theorem to the case $n=2$, with startling alternative conclusions: either g is linear fractional, or it is purely singular. In this context, Kuusalo [6] obtained two similar results under weaker hypotheses, though by "singular" he did not mean quite as strongly singular as Mostow. For example, he proved that if $\mathbf{U}^2/\Gamma$ is of class O_{HB} (no bounded nonconstant harmonic functions), then g is absolutely continuous or singular, and if $\mathbf{U}^2/\Gamma$ is of class O_G (no Green's function), then g is linear fractional or singular.

In another direction, three alternative limit sets for the group Γ have been considered, which I will refer to as the topological, horocyclic, and conical limit sets, and denote respectively by Λ_T, Λ_H, Λ_C. Because Γ acts discontinuously in $\mathbf{U}^n$, these sets all lie in $\bar{\mathbf{R}}^{n-1}=\partial\mathbf{U}^n$, and we have the inclusions $\Lambda_T\supseteq\Lambda_H\supseteq\Lambda_C$. The condition that $\bar{\mathbf{R}}^{n-1}\setminus\Lambda_H$ have measure zero corresponds to the class O_{HB}, and the condition that $\bar{\mathbf{R}}^{n-1}\setminus\Lambda_C$ have measure zero corresponds to the class O_G. Groups with the latter property are precisely the same as groups of "divergence type", where this term traditionally refers to groups for which a certain series (3.6) diverges. The present paper has as its main objective, to show that Mostow's rigidity theorem applies to such groups.