On the local solvability and the local integrability of systems of vector fields

by

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Introduction

The *local solvability* of a first-order linear partial differential equation depends on whether it satisfies the so-called Condition (P) (see [4]). Suppose that the differential operator under study is a complex vector field L, nowhere zero, in some open subset of $\mathbf{R}^{n+1}$. If L is *locally integrable,* that is to say, if in the vicinity of every point the homogeneous equation $Lh=0$ has n independent, and smooth, solutions, one can use them to formulate (P) (see [5]). In the case $n=1$, i.e., when L is defined in an open subset Ω of the plane, there is essentially only one such solution (if one exists at all), in the sense that the differential of any other one is collinear to its differential. Call Z such a solution, and view it as a map $\Omega\to\mathbf{C}$. Condition (P) is equivalent to the property that, locally speaking, the pre-images of points under the mapping Z are *connected.*

But it must be emphasized that the local integrability of L is by no means automatic. In his *"Lectures on linear partial differential equations"* (Reg. Conf. Series in Math., No 17 Amer. Math. Soc. 1973). L. Nirenberg has given the example of a

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