Acta Math., 191 (2003), 225–257
© 2003 by Institut Mittag-Leffler. All rights reserved

Universal properties of $L(\mathbf{F}_\infty)$ in subfactor theory

by

SORIN POPA and DIMITRI SHLYAKHTENKO

University of California Los Angeles, CA, U.S.A.

University of California Los Angeles, CA, U.S.A.

1. Introduction

Let $N \subset M$ be an inclusion of type II_1 von Neumann factors with finite Jones index. Let $N \subset M \subset M_1 \subset ...$ be the associated tower of factors that one gets by iterating the Jones basic construction [J1]. The lattice of inclusions of finite-dimensional algebras $M_i' \cap M_j$ obtained by considering the higher relative commutants of the factors in the Jones tower, endowed with the trace inherited from $\bigcup M_j$, is a natural invariant for the subfactor $N \subset M$.

A standard lattice $\mathcal{G}$ is an abstraction of such a system of higher relative commutants of a subfactor [P3]. That is to say, the relative commutants of an arbitrary finite index inclusion of II_1 factors satisfy the axioms of a standard lattice and, conversely, any standard lattice $\mathcal{G}$ can be realized as the system of higher relative commutants of some subfactor that can be constructed in a functorial way out of $\mathcal{G}$ (see [P3]).

The abstract objects $\mathcal{G}$ carry a very rich symmetry structure. They can be viewed as Jones' planar algebras [J2]. They can also be viewed as group-like objects, serving as generalizations of finitely generated discrete groups and large classes of Hopf algebras and quantum groups.

Along these lines, a subfactor $N \subset M$ can be viewed as encoding an "action" of the group-like object $\mathcal{G} = \mathcal{G}_{N \subset M}$. Given $\mathcal{G}$ it is thus important to understand whether or not it can "act" on a given II_1 factor M; i.e., whether $\mathcal{G}$ can be realized as $\mathcal{G}_{N \subset M}$ for some subfactor N of the given algebra M.

The functorial construction of a subfactor $N \subset M$ with a given standard lattice obtained in [P3], as well as the one preceding it [P1], used amalgamated free products and also depended on a choice of an algebra Q taken as "initial data". However, it remained an open problem whether one can construct a "universal" II_1 factor M that would contain subfactors with any given standard lattice as higher relative commutants, i.e., a factor M on which any $\mathcal{G}$ can "act". It also remained an open problem to identify