

# A new iterative method in Waring's problem

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## 1. Introduction

As is usual in Waring's problem we take  $G(k)$  to be the smallest number  $s$  such that every sufficiently large natural number is the sum of at most  $s$   $k$ th powers of natural numbers.

In this memoir we introduce a new iterative process to Waring's problem. We are thereby able to improve all previous upper bounds for  $G(k)$  when  $k \geq 5$ .

Hitherto the best upper bounds for  $G(k)$  for smaller  $k \geq 4$  have been obtained by variants of the iterative method of Davenport (see [D3], [T3], [Va4] and [Va5]).

When  $5 \leq k \leq 8$  we obtain

**THEOREM 1.1.** *We have  $G(5) \leq 19$ ,  $G(6) \leq 29$ ,  $G(7) \leq 41$ ,  $G(8) \leq 58$ .*

This may be compared with the respective bounds 21, 31, 45, and 62 contained in [Va4] and [Va5].