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A new iterative method in Waring's problem

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Contents

1. Introduction

As is usual in Waring's problem we take $G(k)$ to be the smallest number s such that every sufficiently large natural number is the sum of at most s kth powers of natural numbers.

In this memoir we introduce a new iterative process to Waring's problem. We are thereby able to improve all previous upper bounds for $G(k)$ when $k\geqslant 5$.

Hitherto the best upper bounds for $G(k)$ for smaller $k\geqslant 4$ have been obtained by variants of the iterative method of Davenport (see [D3], [T3], [Va4] and [Va5]).

When $5\leqslant k\leqslant 8$ we obtain

THEOREM 1.1. *We have* $G(5)\leqslant 19$, $G(6)\leqslant 29$, $G(7)\leqslant 41$, $G(8)\leqslant 58$.

This may be compared with the respective bounds 21, 31, 45, and 62 contained in [Va4] and [Va5].