Acta Math., 187 (2001), 139–160

Polynomial inverse images and polynomial inequalities

by

VILMOS TOTIK

Bolyai Institute
Szeged, Hungary

and

University of South Florida
Tampa, FL, U.S.A.

Dedicated to Ronald A. DeVore on his 60th birthday

1. Introduction

Let $\|\cdot\|_E$ denote the supremum norm on the set E. Two of the most used inequalities for the derivatives of polynomials are the Bernstein inequality

$$|P_n'(x)| \leqslant \frac{n}{\sqrt{1-x^2}} \|P_n\|_{[-1,1]}, \quad x \in [-1,1],$$

and the Markoff inequality

$$\|P_n'\|_{[-1,1]} \leqslant n^2 \|P_n\|_{[-1,1]},$$

valid for polynomials P_n of degree at most n. In this paper we are primarily interested in what form these inequalities take on several intervals. We shall see that the extension to general sets involves the equilibrium measure of these sets. We shall give the precise form of the Bernstein inequality for arbitrary compacts, and an asymptotically best form of the Markoff inequality for sets consisting of finitely many intervals. Actually, in this case we shall prove different Markoff inequalities one-one-associated with each one of the endpoints of the system of intervals.

The proofs will heavily use sets that are obtained as the inverse images of intervals under (special) polynomial mappings. We shall see that the original Bernstein and

This research was supported in part by the National Science Fundation, DMS-9801435, and by the Hungarian National Science Foundation for Research, T/022983.