SPECIAL FUNCTIONS ON LOCALLY COMPACT FIELDS

BY

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§ 1. Introduction

In this paper we establish a variety of facts relating to analysis on a locally compact, totally disconnected, non-discrete field, referred to hereafter as K. These fields, which include the $\mathfrak{p}$-adic number fields and their finite algebraic extensions, have been studied in some detail in relation to algebraic number theory and, more recently, in the subject of group representations ([2], [3], [5], [7]).

In these studies certain "special functions" arise as Fourier transforms of additive or multiplicative characters (and combinations of them). The usual approach has been to truncate the characters so as to produce L^1 functions on the additive structure (K^+, dx), or the multiplicative structure (K^*, d^*x) of K and then work with the transforms of the truncated characters.

In [3], however, the non-truncated characters are used in an essential way. It is our purpose to examine the transforms of the non-truncated characters in somewhat more detail than that to be found in [3]. The "special functions" that arise here are the gamma, beta, and Bessel functions. These functions coincide with those introduced in [3]. We also treat the Hankel transform which is not mentioned explicitly in [3]. The gamma, beta, and Bessel functions are first introduced as complex valued functions on appropriate domains and these functions are then related to various distributions on (K^+, dx), (K^*, d^*x) and $(\hat{K}^*, d\pi)$, the group of unitary (multiplicative) characters on K^*.

This paper will be followed with applications to the representations of $SL(2, K)$ by the former author, and to the study of potential spaces and Lipschitz spaces on the finite dimensional vector spaces over K by the latter author.

In § 2 basic harmonic analysis on K^+ and K^* is treated, mostly without proof, but in a form required in the later sections. The major portion of the results stated are either well-known or may be found in [3]. We supply the proof of a few of the less well-known results.

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