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Completeness of translates in weighted spaces on the half-line

by

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1. Introduction

Imagine a one-dimensional monochromatic film, infinitely extended along a straight line, and a one-point signal emitter attached to an infinite rail running parallel to the film, which sends light signals to the film. An emitted signal is recorded on the film, and we may think of the result as a real-valued Borel measurable function on the line. The recording process is assumed reversible, in the sense that if a signal is received, and afterwards the opposite signal is received, the net result is zero. We may move the emitter freely along the rail, and there is a volume dial on the emitter, which permits us to vary the amplitude of the signal, and even reverse it. Suppose the emitter is equipped with a single signal. A natural question is what kind of images can be obtained if the emitter is placed in several positions along the rail and the signal, modified by adjusting the volume dial, is emitted from each of these positions. An interesting subquestion is that of determining which signals may be used to approximate every conceivable image.

When we translate this model to a mathematical setting, we need to define the distance between recorded images. The usual way would be to take the square root of the integral along the film of the square modulus of the difference of the two images, that is, the L^2 metric. Should the sensitivity of the film not be homogeneous, a weight function can be used to express the degree of inhomogeneity.

The above described model is a physical interpretation of translation invariance in function spaces on the real line. This area was initiated in the early 1930's by Norbert Wiener, Arne Beurling, Izrail Gel'fand, and Laurent Schwartz, and a multitude of beautiful papers were produced by them and their followers between 1930 and, say, 1960.

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