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Homoclinic tangencies for hyperbolic sets of large Hausdorff dimension

by

JACOB PALIS and JEAN-CHRISTOPHE YOCCOZ

Instituto de Matemática Pura e Aplicada
Rio de Janeiro, Brazil

Université de Paris-Sud
Orsay, France

Introduction

A fundamental concept in dynamics of a nongradient character is that of a homoclinic orbit, introduced by Poincaré in 1890 [P]: an orbit of intersection at large of the stable and unstable manifolds of a periodic saddle point. It is well known that when such an orbit is *transversal*, it must be accumulated by periodic saddles of the same index (dimension of the stable manifold) as the original saddle with respect to which the homoclinic orbit is doubly asymptotic, as shown by Birkhoff in two dimensions and Smale in general [Bi], [S]. In fact, in this last reference it was proved that transversal homoclinic orbits are always part of a hyperbolic Cantor set, a *horseshoe*, in which the periodic points are dense.

More recently, it has been realized that the creation and unfolding of a *homoclinic tangency*, say for a locally dissipative surface diffeomorphism, gives rise to a striking number of intricate and highly relevant dynamic phenomena: cascades of period doubling bifurcations [YA], infinitely many sinks [N], [R], [PT3], strange attractors of Hénon type [BC], [MV], and hyperbolic Cantor sets combined or not with the previous elements [NP], [PT1], [PT2]. Also, surface diffeomorphisms exhibiting a homoclinic tangency are certainly quite common among nonhyperbolic maps, i.e. maps whose limit set is not hyperbolic. Conjecturally, these homoclinically bifurcating diffeomorphisms may even be dense in the interior of the nonhyperbolic ones, which has turned out to be the case for C^∞ surface diffeomorphisms but in C^1 topology [AM].

Therefore, it seems to us that an important task in dynamics is to unfold the diffeomorphisms exhibiting a homoclinic tangency through k-parameter families and to inquire which of the above or other phenomena are more *common* or *prevalent* in terms of the Lebesgue measure in the parameter space. The main result in the present paper represents a contribution to such a program. Let us first explain it in a more informal