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Abundance of strange attractors

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1. Introduction

In 1976 [He] Hénon performed a numerical study of the family of diffeomorphisms of the plane $h_{a,b}(x,y)=(1-ax^2+y,bx)$ and detected for parameter values $a=1.4$, $b=0.3$, what seemed to be a non-trivial attractor with a highly intricate geometric structure. This family has since then been the subject of intense research, both numerical and theoretical, but its dynamics is still far from being completely understood. In particular one could not exclude the possibility that the attractor observed by Hénon were just a periodic orbit with a very high period.

Recently, in a remarkable paper [BC2], Benedicks and Carleson were able to show that this is not the case, at least for a positive Lebesgue measure set of parameter values near $a=2$, $b=0$. More precisely, they showed that if $b>0$ is small enough then for a positive measure set of a-values near $a=2$ the corresponding diffeomorphism $h_{a,b}$ exhibits a strange attractor. Their argument is a very creative extension of the techniques they had previously developed in [BC1] for the study of the quadratic family on the real line and no doubt it will be important for the understanding of several other situations of complicated, nonhyperbolic dynamics.

When acquainted in 1985 with the work by Benedicks and Carleson, then in progress, Palis suggested that one should in this context think of the Hénon family as a particular, although important, model for the creation of a horseshoe and that the emphasis should be put on the occurrence of unfoldings of homoclinic tangencies. He proposed that the correct setting for Benedicks–Carleson's results is within this more general framework of homoclinic bifurcations and stated the following

Conjecture. Generic one-parameter families of surface diffeomorphisms unfolding a homoclinic tangency exhibit strange attractors or repellers in a persistent way in the measure-theoretic sense (i.e. for a positive measure set of values of the parameter).

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